Mirror Symmetry and Algebraic Geometry

by David A. Cox and Sheldon Katz

Errata for the *first* printing as of January 29, 2020:

Page xviii: Add the following at the bottom of the page: "PS: A list of typographical errors for the book can be found at the web site

`https://math.illinois.edu/~katz/msag/`"

Page 16, first display: In the summation, replace "$d = 1$" with "$d = 0$"

Page 20, three lines below (2.8): Replace "$y_1 = \int_{\gamma_0} \Omega$" with "$y_1 = \int_{\gamma_1} \Omega$"

Page 26, bottom display: In the first line of the display, replace "$770te^{t_1}$" with "$770t_1e^{t_1}$"

Page 32, third bullet on lines -19 and -18: One comment to add here is that for a toric variety X, the implication "X is simplicial $\Rightarrow$ X is an orbifold" is elementary, while the converse "X is an orbifold $\Rightarrow$ X is simplicial" is deeper. A proof can be found in *Rational smoothness and fixed points of torus actions* by M. Brion, Transformation Groups **4** (1999), 127–156.

Page 34, lines 9 and 10: Replace $k\Delta$ with $k\Delta \cap M$ (twice).

Page 34, line 11: Replace $l\Delta$ with $l\Delta \cap M$ and $(k+l)\Delta$ with $(k+l)\Delta \cap M$.

Page 40, line 13: Replace "$\lambda_{n+1}/(D_{v_{n+1}} \cdot C_\sigma)$" with "$(D_{v_{n+1}} \cdot C_\sigma)/\lambda_{n+1}$"

Page 54 line 12: Replace "has at most canonical singularities" with "has at most canonical singularities, as in the proof of"

Page 59, immediately after (4.9), add the sentence: "In (4.9) $H^{n-1}(Z_f, \mathbb{C})$ has a canonical mixed Hodge structure and $Gr^W_{n-1}H^{n-1}(Z_f, \mathbb{C})$ as usual denotes the associated graded piece of the weight filtration with a pure Hodge structure of weight $n-1$."

Page 61, line 5 of Section 4.2: Replace "$\mathbb{R}^5$" with "$\mathbb{R}^4$"

Page 79, line 9: Replace "boundary point of S" with "boundary point of S at which the Gauss-Manin connection has a regular singularity (this is automatic if the family of threefolds extends to a flat family over the boundary point)"

Page 79, line 10: Replace "$\mathcal{F}^0$" with "$\mathcal{F}^3$"

Page 93, line -2: Replace "$(\Delta \cap M) \times \{1\})$" with "$(\Delta \cap M) \times \{1\}$"

Pages 98–99: Proposition 5.5.4 is incorrect as stated. Below you will find numerous changes which fix the statement of the proposition on page 98. Also, parts of the proof on page 99 are seriously flawed. Fixing these would require considerable additions. For this reason, the fixes given below prove only a special case of the result. A complete proof can be found at the web site mentioned in the errata to page xviii.

Page 98, line 8: In (5.43), replace "$\lambda_{r+i}^{c_1(\mathcal{L}_i)\cdot\beta_j}$" with "$(-\lambda_{r+i})^{c_1(\mathcal{L}_i)\cdot\beta_j}$"

Page 98, lines -2 and -1: Replace Proposition 5.5.4 with the following:

PROPOSITION 5.5.4. *The formal function* $(\prod_{i=1}^{k} \lambda_{r+i}^{-1})\,\tilde{I}$ *satisfies the* $\mathcal{A}$*-system associated to* (5.42) *with* $\hat{\beta} = (0, \ldots, 0, -1, \ldots, -1) \in N \times \mathbb{Z}^k$.

Page 99, line 2: Replace "As we discussed in Section 5.5.2, the" with "When $\hat{\beta} = \vec{0}$, by Section 5.5.2, the"

Page 99, line 5: Add the following new sentence: "Then, if we switch to the Z_i for $\hat{\beta} = (0, \ldots, 0, -1, \ldots, -1)$, it follows that the $Z_i((\prod_{i=1}^{k} \lambda_{r+i}^{-1})\tilde{I}) = 0$.

Page 99, line 7: Immediately before the sentence "It follows that ...", insert the following new sentence: "As noted above, we have $c_1(\mathcal{L}_i) \cdot \beta \geq 0$ for all i, and for simplicity, we will also assume that $D_\rho \cdot \beta \geq 0$ for all ρ."

Page 99, line 9: Replace (5.47) with the following new display:

$$\Box_\beta = \prod_{\rho} \partial_\rho^{D_\rho \cdot \beta} - \prod_{i=1}^{k} \partial_{r+i}^{c_1(\mathcal{L}_i)\cdot\beta}, \tag{5.48}$$

Page 99, line −10: Replace the formula for $\Box'_\beta$ with the following

$$\Box'_\beta = \prod_{\rho} \lambda_\rho^{D_\rho \cdot \beta} \quad \prod_{i=1}^{k} \lambda_{r+i} \quad \Box_\beta \quad \prod_{i=1}^{k} \lambda_{r+i}^{-1}$$

Page 99, lines −8 to −5: Replace display (5.51) with the following:

$$\begin{aligned} \Box'_\beta \;=\; & \textstyle\prod_\rho \delta_\rho(\delta_\rho - 1) \cdots (\delta_\rho - D_\rho \cdot \beta + 1) \\ & -q^\beta \textstyle\prod_{i=1}^{k} (-1)^{c_1(\mathcal{L}_i)\cdot\beta} \prod_{i=1}^{k} (\delta_{r+i} - 1) \cdots (\delta_{r+i} - c_1(\mathcal{L}_i) \cdot \beta) \end{aligned}$$

Page 100, lines −3 to −1. Delete these lines.

Page 101, line 4: Delete "It follows that ... (5.37)" and replace with the following

This is a function of q_1, which by (5.43) is $-z$, where z is defined in (5.36). It follows that $y_0, \ldots, y_4$ are solutions of the fifth order GKZ equation (5.37), provided we replace z with $-q_1$. The sign difference between q_1 and z will be discussed in Section 6.3.3.

Page 106, line 1: Replace "$\nabla(e_{3-p})$" with "$\nabla_\delta(e_{3-p})$"

Page 106, immediately after the display on line 21: Add the new sentence "Due to a constant of integration, the above expression for q is only defined up to a multiplicative constant. We fix this choice by requiring that q is of the form $q = z\exp(f(z))$, where $f(z)$ is holomorphic with $f(0) = 0$."

Page 137, line −6: Replace "induces isomorphism" with "induces an isomorphism"

Page 138, line 11: Insert the new sentence "However, [**Szendroi2**] shows that Conjecture 6.2.8 is false in general."

Page 138, line below (6.24): Replace "Strictly speaking, we should take the quotient of this by the automorphisms of V. However, $\mathrm{Aut}(V)$" with "We should take the quotient of this by $\mathrm{Aut}_{\mathrm{toric}}(V)$, the subgroup of automorphisms of V which preserve $H^2_{\mathrm{toric}}(V)$. Then $\mathrm{Aut}_{\mathrm{toric}}(V)$"

Page 138, 3 lines below (6.24): Replace "$\mathrm{Aut}(V)$" with "$\mathrm{Aut}_{\mathrm{toric}}(V)$"

Page 146, line -26: Replace "$D_6 \sim D_1 - 2D_2$" with "$D_6 \sim D_3 - 2D_1$"

Page 147, line 20: Replace "has rank 2" with "has rank 4"

Page 170, line 1: Replace "stacks are" with "the stacks of interest to us are"

Page 170, line 2: Replace "to the category of sets" with "to the category of groupoids"

Page 170, line 3: After "as we go along.", insert the new sentence "For simplicity of exposition, we will abuse terminology by refering to stacks as if they were functors from schemes to sets."

Page 172, three lines above (7.11): Replace "$\mathbb{P}_1$" with "p_1"

Page 183, display in part (ii) of Definition 7.1.9: Replace "$\cup \ldots \cup$" with "$\cup \cdots \cup$"

Page 184, line 3 of Section 7.2: Replace "J-holomorphic maps in symplectic geometry" with "J-holomorphic maps in symplectic geometry, due to [**Gromov**],"

Page 193, line 2: Replace "$[X] \in H^0(X, \mathbb{Q})$ is the fundamental class of X" with "$1_X = [X] \in H^0(X, \mathbb{Q})$ is the Poincaré dual of the fundamental class of X"

Page 193, line 15: Replace "π_{n*}" with "$\pi_{n!}$"

Page 194, line 4: Replace "q_{n_2+1}" with "q_1"

Page 194, line 5: Replace "$(C, p_1, \ldots, p_{n_1}, q_1, \ldots, q_{n_2})$" with "$(C, p_1, \ldots, p_{n_1}, q_2, \ldots, q_{n_2+1})$"

Page 194, line 15: Replace "$(C_1 \cup C_2, p_1, \ldots, p_{n_1}, p, p_{n_1+1}, \ldots, p_n, q)$" with "$(C_1 \cup C_2, p_1, \ldots, p_{n_1}, p, q, p_{n_1+1}, \ldots, p_n)$"

Page 194, line 2 of the Reduction Axiom: Replace "Δ_a" with "T_a"

Page 194, line 5 of the Reduction Axiom: Replace "ϕ" with "ψ"

Page 197, line 11: Replace "$\overline{M}_{0,n}(X, \beta)$" with "$\overline{M}_{0,3}(X, \beta)$"

Page 197, line 12: Replace "$2 \dim X + 2$" with "$4 \dim X + 2$"

Page 201, line -7: Replace "Fundamental Class Axiom" with "Point Mapping Axiom"

Page 202, line 5 after Conjecture 7.4.3: Replace "are all are" with "are all"

Page 203, equation (7.48): In the summation, replace "$d = 1$" with "$d = 0$"

Page 205, line -11: replace "$d = 2$" with "$k = 2$"

Page 206, line 18: A fully rigorous proof of the enumerative significance of $17,601,000$ can be found in *Node polynomials for families: methods and applications* by S. Kleiman and R. Piene, Math. Nachr. **271** (2004), 69–90.

Page 206, equation (7.54): The Clemens conjecture for $d = 10$ is proved in *Rational curves of degree 10 on a general quintic threefold* by E. Cotterill, Comm. in Algebra, **33** (2005), 1833–1872. This, plus the paper mentioned in the previous erratum, give a complete proof that $n_{10} = 6 \times 17,601,000+$ number of rational curves of degree 10 on the quintic threefold.

Page 207, line 5, replace "$17,601,600$" with "$17,601,000$"

Page 218, line 16 (this is the line following the second display): Replace "$\int_V T^i \cup T_j = \delta_{ij}$" with "$\int_X T^i \cup T_j = \delta_{ij}$"

Page 233, line -4: replace "commutative" with "supercommutative"

Page 235, line -18: Replace "coefficient of q^β in $T_i * T_j$ has degree $\deg(T_i)+\deg(T_j)+2\int_\beta \omega_X$, just as we noted for the small quantum product in the proof of Proposition 8.1.5." with "coefficient of $T^\ell q^\beta$ in $T_i * T_j$ has degree $\deg(T_\ell) - (\deg(T_i) + \deg(T_j) + 2\int_\beta \omega_X)$. The argument is similar to the proof of Proposition 8.1.5."

Page 240, line -3 to -5: Replace this paragraph with the following: "The potential function Φ we studied earlier satisfies the conditions of Proposition 8.4.1. This is why the big quantum product makes $H^*(X, \mathbb{C})$ into a Frobenius algebra."

Page 244, line -2: Replace "q^β is defined as in (8.40) to be" with "$q^\beta = e^{2\pi i \int_\beta \omega}$, where as in (8.40),"

Page 246, line 17: Replace "exponents are positive" with "exponents in the q_j are positive"

Page 275, line -7: Delete the word "compact"

Page 275, line -6: Delete "with EG,"

Page 275, line -3: Replace "and $EG = \pi_1^* S \otimes \cdots \otimes \pi_n^* S$," with "and the vector bundle obtained from EG by the natural action of $(\mathbb{C}^*)^n$ on $\mathbb{C}^n$ is $\pi_1^* S \oplus \cdots \oplus \pi_n^* S$,"

Page 276, line -15: Replace "such that the action of G lifts to an action of E" with "together with a lifting of the action on X to an action on E"

Page 276, line -10: Replace "$\text{Euler}_T(E)$" with "$\text{Euler}_G(E)$"

Page 276, line -5: Replace "$\mathcal{O}(\lambda_1) \oplus \cdots \oplus \mathcal{O}(\lambda_1)$" with "$\mathcal{O}(\lambda_1) \oplus \cdots \oplus \mathcal{O}(\lambda_n)$"

Page 277, line 3: Replace "smooth manifold" with "compact complex manifold"

Page 277, line 14: Replace "assertion" with "assertions"

Page 277, line -3: Replace "$H_T(\mathbb{P}^r)$" with "$H_T^*(\mathbb{P}^r)$"

Page 278, line 3: Replace "$c_r(E_T^*)$" with "$(-1)^r c_r(E_T^*)$"

Page 278, line after (9.5): Replace "is has" with "has"

Page 280, first display: replace "Z_i" with "Z_j", "a_i" with "a_j", and "N_i" with "N_j".

Page 283, line -19: replace "(f_{C_v}, C_v)" with "C_v".

Page 285, line -12: Replace "different from e" with "different from v"

Page 291, second display: Replace "$Z^0(U_0 \cap U_1, f_3^* \mathcal{O}_{\mathbb{P}^1}(-1))$" with "$Z^1(U_0 \cap U_1, f_3^* \mathcal{O}_{\mathbb{P}^1}(-1))$"

Page 291, 2 lines above the third display on page: Replace "have of" with "have all of"

Page 293, line −6: The erratum for page 206 gives a reference that rigorously establishes the enumerative significance of $17,601,000$.

Page 304, line −7: The list of axioms for gravitational correlators should also include a Linearity Axiom and an Effectivity Axiom, which are similar to the versions for Gromov-Witten invariants stated on page 192.

Page 305, line −9: Delete "$c_1(\mathcal{L}_i)^{d_i}\cup$" and replace "$\pi_*(\xi)$" with "$\pi_*(\prod_{i=1}^n c_1(\mathcal{L}_i)^{d_i}\cup\xi)$".

Page 305, lines −4 and −1: Delete "(X,β)" once on line −4 and three times on line −1.

Page 306, line 3: replace "τ_1" with "τ_{d_1}".

Page 306, line 4: replace "τ_n" with "τ_{d_n}".

Page 316, first line of display in Theorem 10.2.4: Replace "$S(e^{-T_j}\cup T)$" with "$s(e^{-T_j}\cup T)$"

Page 316, second line of display in Theorem 10.2.4: Replace "$-S(T_j\cup T)$" with "$-s(T_j\cup T)$"

Page 331, line 1: Replace "the the" with "the"

Page 336, line −22 (this is display (11.6)): Replace "$\alpha_n(w_0,w_1))$" with "$\alpha_n(w_0,w_1)))$"

Page 336, lines −20 and −19: Replace "We summarize this by saying that" with "In particular,"

Page 336, line −18: Replace "without common factor." with "without common factor, modulo scalars."

Page 353, lines −11 and −9: Replace "P" and "Q" with "$\hat{P}$" and "$\hat{Q}$" respectively

Page 353, line −3: Replace "h^{-1}" with "$\hbar^{-1}$"

Page 381, second line of display (11.74): In the denominator of the fraction, replace "$\hbar - c_1(\mathcal{L}_1)$" with "$\hbar - c$", just as in display (11.52) on page 365.

Page 394, line −11: Replace "$\hbar\delta_1\delta_2 - z_2$" with "$\hbar\delta_4\delta_2 - z_2$"

Page 398, line 23: Replace "but these techniques do not apply" with "and subsequent work of A. Elezi and B. Kim has shown that these techniques can be applied"

Page 409, line −3: Replace "smooth $(1,1)$-form" with "smooth, closed $(1,1)$ form"

Page 437, index entry [AKl]: Replace "A. Altmann" with "A. Altman"

Page 444, line 11: Insert a new reference:

[Gromov] M. Gromov, *Pseudoholomorphic curves in symplectic manifolds*, Invent. Math. **82** (1985), 307–347.

Page 449, line 1: Replace "by irreducible" with "be irreducible"

Page 449, line −5: Insert a new reference:

[Szendroi2] B. Szendroi, *On a conjecture of Cox and Katz*, Math. Z., **240** (2002), 233–241, math.AG/0110166.