

MATH 510
Riemann Surfaces and Algebraic Curves
Fall 2023
SYLLABUS
Revised 8/28/23 to include an office hour

Time and place: TH 12:30–1:45, 162 Noyes Laboratory

Instructor: Sheldon Katz

Office and hours: 301 Altgeld Hall, Mondays at 2, or by appointment

Instructor contact information: 265-6258, katzs@illinois.edu

Text: R. Miranda, Algebraic curves and Riemann surfaces, Graduate Studies in Mathematics **5**, AMS, 1995

Grader: Alex Pacun, pacun2@illinois.edu

Prerequisite: MATH 542.

Course website: katzs.web.illinois.edu/class/f23/

Course Description:

Compact Riemann surfaces appear in mathematics in a wide variety of ways. In topology, compact Riemann surfaces up to homeomorphism are precisely the compact orientable manifolds of dimension 2. In differential geometry, they are 2 dimensional compact orientable Riemannian manifolds up to conformal equivalence. In complex analytic geometry, they are precisely the compact complex manifolds of dimension 1. In algebraic geometry, they are the smooth projective varieties of dimension 1 over the field of complex numbers. They are also in 1–1 correspondence with a purely algebraic object, the set of fields of transcendence degree 1 over the complex numbers. They also come up in physical applications such as potential theory, conformal field theory, and string theory (alas, not covered in the course due to time constraints).

This course serves as an introduction to Riemann Surfaces from both the algebraic and function-theoretic points of view. The emphasis will be geometric. Topics include projective algebraic curves, differential forms, integration, divisors of poles and zeroes, linear systems, the Riemann-Roch theorem, Serre duality, and applications, and other topics in Chapters 1–8 of the text. The course will also introduce students to algebraic geometry over the complex numbers beginning with the analytic theory, serving as an invitation to algebraic geometry.

Lectures will parallel the topics in the text but the treatment will not always be the same. Students are advised to attend the lectures, read the book, and stay on top of the homework.

Homework: will be assigned, collected in class, and graded. Alternatively, you may leave your homework in the grader's mailbox in the math department mailroom, 250 Altgeld Hall. You can discuss problems with each other and with the instructor, but solutions must be written up individually. You are expected to submit your homework on time (by the due date, either in class or by 5pm on the due date in the grader's mailbox). Late homework will not be accepted under any circumstances after homework has been returned to students (typically within one week of the due date). The homework due dates will be announced in class and posted on the course website. There will be approximately 10 homework assignments during the semester.

Grading: Based on homework assignments and classroom participation