Some Material From Math 511, Introduction to Algebraic Geometry [H, Ch 1]

$k=\bar{k}$ $\quad \mathbb{A}^n = \mathbb{A}^n_k = \{(a_1, \dots, a_n) \mid a_i \in k\}$

$y = x^2$

$I = (y - x^2)$

$t \longmapsto (t, t^2, t^3)$

$I = (y - x^2, z - x^3)$

twisted cubic curve

Morphism $\quad \varphi : \mathbb{A}^1 \longrightarrow \mathbb{A}^3$

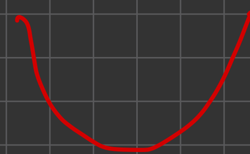
$\varphi(t) = (t, t^2, t^3)$

Regular fcn $\quad X \longrightarrow \mathbb{A}^1$

$xy = 0$

reducible, singular

$y = x^2$



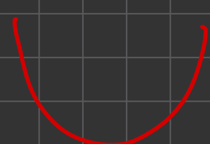
irreducible, a variety

nonsingular, nonsingular variety

$$\mathbb{P}^n = \mathbb{P}^n_k = \{(x_0, \ldots, x_n) \in k^{n+1} - 0\} / x \sim \lambda x$$

$$\lambda \in k^* = k - 0$$

$$\mathbb{A}^n \hookrightarrow \mathbb{P}^n \qquad (x_1, \ldots, x_n) \hookrightarrow (1, x_1, \ldots, x_n)$$



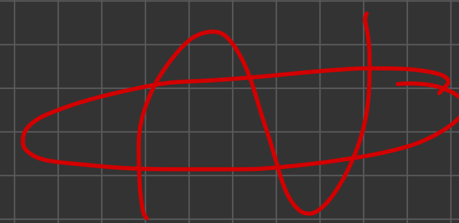
$y = x^2$

$(0,0,1)$

$yz = x^2$

homogeneous

Bézout's Thm: Plane curves of degree d and e with no common components meet in de points, including multiplicity



Generalizes to higher dimension

$$\mathbb{P}^1 \longrightarrow \mathbb{P}^3 \qquad (s,t) \longmapsto (s^3, s^2t, st^2, t^3)$$

(x_0, x_1, x_2, x_3)

$$I = (x_0x_2 - x_1^2,\ x_1x_3 - x_2^2,\ x_0x_3 - x_1x_2)$$

Not a complete intersection — need more than 2 equations. If we use only first two equations, get extraneous line $x_1 = x_2 = 0$, i.e. reducible

Consistent with Bézout: $2 \cdot 2 = 3 + 1$

A major problem in algebraic geometry

Classify all algebraic varieties

Solved for nonsingular curves

Still unsolved for nonsingular surfaces!

Moduli problem